EXPERIMENTAL STUDY OF THE FORCE OF INTERNAL WAVES ACTING ON A STATIONARY SPHERE

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UDC 532.59

There are a large number of theoretical works on the problem of wave loads acting on submerged bodies having simple geometric shapes (see the reviews in [1, 2]). These works concerned mainly the case of a fluid with a uniform density. Surface-wave action on horizontal and vertical cylinders has been studied in greatest detail experimentally. The literature on this problem is reviewed in [3]. Wave loads exerted by surface waves on a submerged sphere were investigated in, for example, [4, 5]. Much less experimental information is available on liquids with a nonuniform density (stratified liquids). Measurements of the resistance of bodies with close to linear stratification was measured experimentally in [6, 7]. The forces exerted by internal waves on elongated bodies with a practical geometry were studied experimentally in [8, 9].

In the present work we investigated experimentally the force exerted by internal waves on a stationary sphere under conditions of close to bilayer stratification for characteristic Reynolds numbers ranging from 10 to 280 and Kelegan–Carpenter number ranging from 0.06 to 0.95.

The experiments were performed in a basin with length l = 6 m, width b = 0.2 m, and depth h = 0.6 m. The basin was filled with two liquids of different density. The top layer consisted of water with density ρ_1 = 0.999 g/cm^3 and the bottom layer consisted of a solution of glycerine in water with density ρ_2 = 1.010 g/cm^3. A diagram of the experimental apparatus is displayed in Fig. 1. Internal waves were generated with the help of a half-cylinder undergoing translational harmonic oscillations along the end wall of the basin. The opposite end of the basin was equipped with a wave damper 6 in the form of a plate sloping at an angle of 6° with respect to the horizontal plane. Two-component balances 1 were used to measure the loads acting on the sphere 3. The forces were transferred with the help of a rod 2 and submerged knife edges 4 to the elastic elements 5 whose deformation was measured with the help of induction displacement sensors. The maximum load in the experiments did not exceed $9\cdot10^{-4}$ N. The elastic displacement of a sphere under the action of such a force did not exceed $4.5\cdot10^{-3}$ mm. The diameter of the sphere d = 4 cm. The minimum characteristic oscillation frequency of the balances with a model suspended on them in water was 2.7 Hz and maximum force frequency in the experiments did not exceed 0.24 Hz.

A stationary rectangular coordinate system Oxy (Fig. 1) was used. The origin of the coordinate system was located beneath the center of the sphere. The x axis is horizontal, and in the unperturbed state of the liquid it is also the line of equal density $\rho_0 = (\rho_1 + \rho_2)/2$, taken as the conventional interface separating the media; the y axis is directed vertically upwards. The Oxy plane is also the vertical symmetry plane of the basin. Internal waves incident on the sphere propagate in the positive x direction. In this coordinate system the depth distribution of the density was approximated well by the relation

$$\rho(y)=\rho_0-0{,}5(\rho_2-\rho_1)\,\mathrm{th}(y/\delta)$$

where δ is a parameter characterizing the thickness of the smeared layer, and ranged in the experiments from 0.42 to 0.58 cm.

The parameters of the internal waves were recorded with the help of a resistive-type wavemeter. The wavemeter was placed on one side of the sphere. This made it possible to judge the phase shifts between oscillations of the wave profile and the forces acting on the sphere. The amplitude of the waves was determined taking into account the dynamical

Novosibirsk. Translated from Prikladnaya Mekhanika i Technicheskaya Fizika, No. 4, pp. 103-107, July-August, 1993. Original article submitted July 3, 1992; revision submitted July 14, 1992.

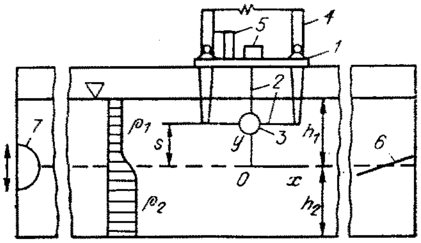

Fig. 1

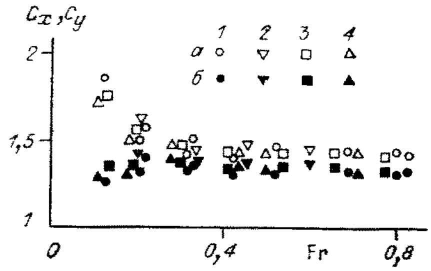

Fig. 2

calibration of the wavemeter. The oscillations of the wave profile and the forces were recorded before the arrival of the reflected waves, though the amplitude of these waves in the measurement zone did not exceed 5% of the amplitude of the incident waves.

In the present problem it is most convenient to make comparisons to the linear theory of waves in an ideal, unbounded, two-layer liquid. A sufficient condition for modeling experimentally infinite depths is

$$(2+\varepsilon)\,\omega^2 h_{1,2}/\varepsilon g > 3, \tag{1}$$

where $\varepsilon = \rho_2/\rho_1 - 1 = 0.011$; $h_{1,2}$ is the depth of the top (bottom) layer. The minimum value of $h_{1,2}$ in the experiments was 0.25 m. Moreover, the parameter $d/h_{1,2}$ must be small. In the experiments this ratio did not exceed 0.16. Calculations within the linear theory of waves [2] show that in the experimental range of values of the parameters of the problem it is sufficient that $d/h_{1,2} \leqslant 0.2$.

The question of the influence of the basin walls has been less studied. Experimental data showing that for a vertical cylinder with $d/b \le 0.18$ (d is the diameter of the cylinder and b is the width of the basin) there is virtually no obstruction of the flow with oscillatory motion are presented in [3]. In our experiments $d/b = 0.2$, but the influence of the walls can apparently be neglected, since a sphere obstructs that cross section of the basin much less than does a cylinder.

The experiments were performed in two series. In the first series the dynamical action of the waves on a sphere in the absence of any effect due to the variable buoyancy force was studied. For this the sphere was placed far enough away from the interface of the media so that the condition $s > a + r + 3\delta$ (s is the distance from the center of the sphere to the x axis, r is the radius of the sphere, and a is the amplitude of the internal waves) was satisfied. The frequency of the waves and the depth s were varied. The results of this series of experiments are presented in Fig. 2. The values of the parameter $Fr = (2 + \varepsilon)\omega^2 r/\varepsilon g$, which in such problems plays the role of Froude's density number, is plotted along the abscissa and the coefficients of the horizontal and vertical forces, defined as

$$C_x = F_{xa}/\rho_1 V \dot{u}_a, \quad C_y = F_{ya}/\rho_1 V \dot{w}_a, \tag{2}$$

are plotted along the ordinate. In Eq. (2) V is the volume of the sphere; $\dot{u}_a = \dot{w}_a = \alpha\omega^2 \exp(-(2 + \varepsilon)\omega^2 s/\varepsilon g)$ is the amplitude of the local accelerations of fluid particles at a depth corresponding to the center of the sphere, as determined from the linear theory of waves in an ideal unbounded two-layer liquid with a jump in density; F_{xa} and F_{ya} are the amplitudes of the horizontal and vertical forces. The symbols in the sets a and b refer to the coefficients C_x and C_y, respectively; the variants 1–4 correspond to relative depths s/d = 1.38, 1.66, 1.81, and 2.09.

The condition for absence of any influence of the bottom and of the free surface (1) holds for $Fr > 0.24$. The values of the coefficient C_x were obtained by analyzing the measurements of the total horizontal force acting on the sphere and the submerged knife edges. The measurements performed in the absence of a sphere showed that the amplitude of the forces acting on the knife edges does not exceed 5% of the amplitude of the total force. Apparently, the knife edges are responsible for the fact that C_x is somewhat greater than C_y. The second possible source of this effect could be the weak irregularity of internal waves in the experiment, as expressed in the variation of amplitudes by ±3%

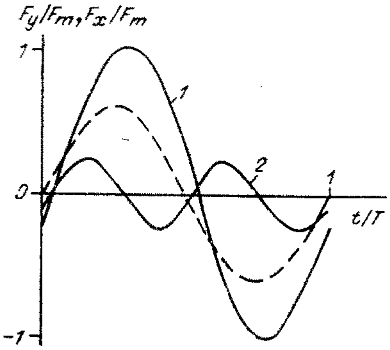

Fig. 3

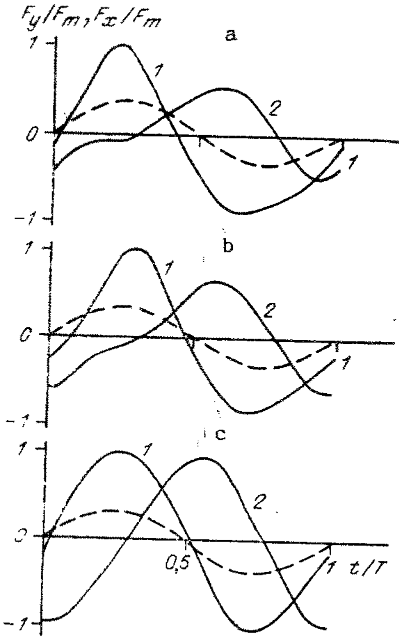

Fig. 4

around the average value. In [4, 10] it is shown that irregular waves reduce the force coefficients below their values in the case of regular waves, the effect being stronger for the coefficient of the vertical force.

It follows from Fig. 2 that C_x and C_y are virtually constant and close to 1.5, i.e., the value predicted by the theory of an ideal liquid for a sphere in an oscillatory flow. The increase in C_x and decrease in C_y for small values of Fr are due to approximate kinematics of liquid particles in Eq. (2) used for normalization. The amplitudes $\dot{u}_a$ and $\dot{w}_a$ were estimated according to a theory that takes into account the finiteness of the depths h_1 and h_2. In this case C_x and C_y were found to be constant in the entire range of experimental values of Fr.

The relation (2) with constant C_x and C_y presumes that the loads from a wave of unit amplitude have for a prescribed depth of the sphere a maximum at $\omega_* = \sqrt{\varepsilon g/(2+\varepsilon)s}$. This relation was satisfied in the experiments to within 1%.

The pattern of phase shifts between the internal waves and the oscillations of the vertical and horizontal forces shows that the force interaction is mainly determined by the inertial component. The phase shift is $\psi_1 = 90° + \varphi_1$ for the horizontal force and $\psi_2 = 180° + \varphi_2$ for the vertical force. The increments arising as a result of the effect of viscosity $\varphi_1 = 9 \pm 2°$ and $\varphi_2 = 18 \pm 2°$ are small and are virtually independent of the frequency.

The Kelegan–Carpenter number, defined as $K_c = u_a T/d$ (u_a is the amplitude of the velocity of a liquid particle at a depth corresponding to the position of the center of the sphere and T is the period of the oscillations), varied from 0.06 to 0.95 and Reynolds number $Re = u_a d/\nu$ varied from 10 to 280.

In such problems the forces are separated into inertial and damping components with the help of Morrison's equation. In the component form this equation is [4]

$$F_x = \frac{1}{8} C_{dx} \rho \pi d^2 u \sqrt{u^2 + w^2} + \frac{1}{6} C_{mx} \rho \pi d^3 \dot{u},$$

$$F_y = \frac{1}{8} C_{dy} \rho \pi d^2 w \sqrt{u^2 + w^2} + \frac{1}{6} C_{my} \rho \pi d^3 \dot{w},$$

where u and w are the horizontal and vertical components of the velocity of the liquid particles; $\dot{u}$ and $\dot{w}$ are the local accelerations; C_{dx} and C_{dy} are the coefficients of the damping forces; and C_{mx} and C_{my} are the coefficients of the inertial forces.

Equations (2), together with information on the phase shifts, give the following relations:

$$C_x \cos(\omega t - \varphi_1) = \frac{3}{8\pi} C_{dx} K_c \sin \omega t + C_{mx} \cos \omega t,$$

$$C_y \sin(\omega t - \varphi_2) = -\frac{3}{8\pi} C_{dy} K_c \cos \omega t + C_{my} \sin \omega t.$$

The experimental results are described well by Morrison's equation with $C_{mx} = C_x \cos \varphi_1$, $C_{my} = C_y \cos \varphi_2$ and the relations $C_{dx} = 8\pi C_x \sin \varphi_1/3K_c$, $C_{dy} = 8\pi C_y \sin \varphi_2/3K_c$ ($C_{mx} = 1.43$, $C_{my} = 1.26$, $C_{dx} = 1.7/K_c$, $C_{dy} = 3.44/K_c$). These values of the coefficients agree well with the results obtained for regular surface waves [4, 5].

The forces acting on a sphere located in the pycnocline were measured in the second series of experiments. In this case, the buoyancy force was stronger than the inertial force and the vertical force varied almost in-phase with the oscillations of the interface separating the media. If the sphere was located precisely at the center of the pycnocline, i.e., s = 0, then frequency doubling, compared with the frequency of the incident waves, is observed for the horizontal force. This effect is explained by the balance of the inertial forces on the sphere with nearly equal densities of the top and bottom layers. An example of the records of such a process is displayed in Fig. 3. The dimensionless time is plotted along the abscissa as a fraction of the period of the oscillations and the values of the vertical and horizontal forces plotted along the ordinate (curves 1 and 2, respectively); the forces are dimensionless and measured in units of the maximum value of the vertical force over a period of the oscillations F_m ($F_m = 9.1 \cdot 10^{-4}$ N, a = 1.1 cm, $\delta = 0.48$ cm, Fr = 0.34). The dashed line is the wave profile.

If the center of the sphere is displaced somewhat from the line of equal density ρ_0, then the time dependence of F_x and F_y is complicated, both the amplitude and the form of the force action itself changing with increasing frequency of the waves. An example of such evolution with relative depth of the sphere s/d = 0.26 and thickness of the smeared layer δ = 0.51 cm is displayed in Fig. 4. All notations are similar to those employed in Fig. 3; the relative wave amplitude a/d = 0.255; 0.275; 0.225, $F_m = 6.88 \cdot 10^{-4}$; $6.88 \cdot 10^{-4}$; $2.84 \cdot 10^{-4}$ H, Fr = 0.255; 0.349; 0.525 for variants a–c, respectively. The traces displayed in Fig. 4 indicate that the nonlinearity has a significant effect, and the nonlinear effects have a tendency to weaken with increasing frequency.

I thank E. M. Romanov for providing extensive assistance in the experiments and V. I. Bukreev and I. V. Sturov for assistance in formulating the problem and for a discussion of the results.

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